Supplementary Information for

Title: Learning and inference using complex generative models in a spatial localization task

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**Figure S1**: Comparing participants’ weights in Experiment 2 to weights assigned by an ideal observer with perfect knowledge of the prior distributions. Weights assigned by an ideal observer with perfect knowledge of the broad (A) and narrow (B) priors, in the low (blue), medium (green) and high (red) likelihood variance conditions are depicted as dashed horizontal lines. In both prior conditions, participants’ empirical weights in Experiment 2 moved closer to the ideal weights as a function of exposure to the task. By the final temporal bin, participants’ weights showed the same qualitative trends as the ideal observer, but differed quantitatively in a manner that interacted with prior and likelihood variance. The pattern of deviations from ideal weights, as a function of exposure, is consistent with participants starting with uniform internal estimates for the priors and refining these internal estimates as a result of exposure, but not attaining the perfect knowledge of the priors assumed by the ideal observer in this analysis, even by the final temporal bin. Columns represent means and error bars represent SEM across participants.
Supplementary Figure S2

Figure S2: Examining the influence of increased exposure to the prior distributions. In both the broad (A) and narrow (B) prior conditions, participants’ empirical weights in Experiment 3 moved closer to the weights predicted by an ideal observer that had perfect knowledge of the prior distributions, as a function of exposure to the task. By the final temporal bin, when participants in each group had been exposed to double the number of samples from the relevant prior distribution, their weights were closer to the ideal weights, than participants’ weights in the final temporal bin of Experiment 2. Columns represent means and error bars represent SEM across participants.
**Experiment 3: Increasing exposure to prior distributions**

Across two experiments (see main text), we found that human observers can learn complex generative models in a spatial localization task, and integrate this learned knowledge with uncertain sensory information in a manner consistent with Bayes-optimal behavior. Comparing participants’ empirical weights in Experiment 2, to the ideal weights that would be assigned by a Bayes-optimal observer that had perfect knowledge of the underlying generative model, we found that participants’ empirical weights moved closer to the ideal weights as a function of exposure to the task. However, by the end of the experiment, they differed quantitatively from the ideal weights in a manner that interacted with prior and likelihood variance (Figure S1). This pattern suggests that even by the end of the experiment they might not have received enough exposure to have attained the perfect knowledge of the prior distributions assumed by the ideal observer.

To test this possibility, we ran a third experiment in which we doubled the number of trials, with respect to each underlying prior distribution, that participants were exposed to. To simultaneously maintain the same duration for this experiment, as in Experiment 2, we recruited two groups of participants (N=8 in each group), and exposed each group to 1600 trials in which the target was drawn from one of the two underlying Gaussian distributions used in Experiment 2. Each group of participants was therefore exposed to twice the number of samples from the relevant prior distribution. The 1600 trials carried out by each group were split up into individual likelihood conditions, and each condition was further split into 8 temporal bins (so as to include the same number of trials in each temporal bin, as in Experiment 2).

**Methods**

*Participants:* Sixteen undergraduate students at the University of Rochester participated in this experiment, in exchange for monetary compensation. Each participant had normal or corrected-to-normal vision, was naïve to the purpose of the study, provided informed written consent and did not participate in Experiments 1 and 2. The University of Rochester’s institutional review board approved all experimental protocols.

*Procedure:* Before the start of the experiment, participants were provided with task instructions that were nearly identical to those provided to participants in Experiment 2.
The only exception was that instead of being told that the bucket might be located on the left side or the right side of the display on each trial, they were told that the bucket would always be located on the left side of the display (for one group of participants) or on the right side of the display (for the second group of participants). Each group carried out a total of 1600 trials, split evenly across 4 experimental blocks. Short breaks were allowed between blocks and the total experimental duration, including breaks, was an hour.

Data Analysis: As in Experiments 1 and 2, in the trials in which sensory information was available, we used linear regression to compute the weights assigned to the centroid of the cluster of dots (the likelihood) and to the mean of the underlying target distribution (the prior). On the trials in which no sensory information was available, we computed participants’ mean responses, across all trials in each temporal bin. As in Experiments 1 and 2, we again focused on performance in the vertical dimension.

Results and Discussion

Across the two groups of participants, we found that the weights assigned by participants to the sensory information in the final temporal bin were significantly closer to the ideal weights predicted by the Bayes-optimal observer that had perfect knowledge of the prior distributions (Figure S2). Specifically, when we consider the medium and low reliability conditions in the final exposure bin, participants’ weights to the sensory information in Experiment 3 were significantly closer to the ideal weights, than participants’ weights in Experiment 2 (Figure S2). Indeed, we found that participants continued to update their weights throughout this experiment, including between trial bin 4 (which was the amount of exposure participants had to each underlying distribution in Experiment 2) and trial bin 8. This pattern of results is consistent with the notion that the quantitative deviation observed between ideal weights and participants’ weights at the end of Experiment 2 was largely driven by participants not having received enough exposure to have attained the perfect knowledge of the prior distributions assumed by the ideal observer.