Figure S1. The scale-space response to an edge is more compact in the $N_3^+$ (or $N_3$) model than the $N_1$ model. This enables the $N_3^+$ model to resolve pairs of closely spaced edges better. Here the 2 edges had the same polarity, amplitude and blur ($\beta = 8$ pixels), with spatial separation of $5\beta$, $3\beta$ or $2\beta$, as shown. The $N_1$ model failed to resolve two peaks for edges separated by less than $4.8\beta$, while $N_3^+$ could resolve them down to $2.5\beta$. This was true for all blurs $\beta$. The psychophysical limit for this task has yet to be tested.
Figure S2. Scale-space response maps for the sine-wave experiment (Figures 4A, 4B, and 4C). Light and dark regions represent positive and negative output values (for the linear N1 model, middle row) and the positive and negative output channels (N3+, N3-) (bottom row). Segregation of positive and negative edge responses in the N3+/- model prevents edge blur coding from being influenced by neighbouring edges.
Figure S3. (A) Test stimulus was a grating formed as the (odd) harmonics of a square-wave grating, up to the $n$th harmonic, where $n = 1, 3, 5 \ldots 15$. Subjects judged the blur of its central edge against a single, Gaussian comparison edge, using the two-interval procedure described in the text. Fundamental frequency $f$ was 0.35 c/deg; fundamental contrast = 0.32. Not surprisingly, as $n$ increased, the closer approximations to a step edge looked sharper (see insets), with excellent quantitative agreement between observers (M.A.G., T.C.A.F.). With no free parameters, all 3 models captured this trend fairly well, but predictions from $N^3+$ were more accurate than the linear $N_1$ or $N_3$ models. (B) Experiment similar to A, but the test grating contained only $f$ and $3f$ components, where $f = 0.33$ or 1 c/deg (upper and lower datasets). For contrast ratios 1, 2, 4, 8, 16, 32, the pairs of ($f$, $3f$) contrasts were (8, 8), (11.3, 5.7), (16, 4), (22.6, 2.8), (32, 2), (45.2, 1.4). As the $f$ component contrast increased (relative to $3f$), the central edge looked increasingly blurred. Both models $N_1$ and $N^3+$ predicted the results fairly accurately, but without the rectifier the N3 model failed badly at relatively low $3f$ contrasts, where contrast ratio > 8. As with pure sine-waves, the rectifier plays a key role in isolating adjacent edges from each other when blur is large relative to separation.
Figure S4. Edges look sharper when they are shorter (cf. Figure 6). These scale-space response maps show how the response peak (representing edge location and blur) shifts to smaller scales as edge length is reduced. The key factor is edge length expressed in units of blur $\beta$. The peak shift occurs for both models $N_1$ and $N_3^+$ (or $N_3$), but $N_1$ over-estimated the experimentally observed shift, while $N_3^+$ was fairly accurate. Here it was assumed that all filter kernels were partial derivatives of a circular Gaussian function.
Figure S5. Shows the influence of the scaling ('normalization') exponent $\alpha$ on the scale-space response to an edge. Columns (left to right) illustrate $\alpha = 0$, $n/4$, $n/2$ and $n$, where $n$ is the derivative order of the filter or channel. Normalization scales the final filter output by the factor $\sigma^\alpha$, where $\sigma$ is the scale of the filter. Hence larger values of $\alpha$ progressively amplify the large-scale filters relative to the smaller ones. This shifts peak responses to the larger scales, but eventually leads (when $\alpha = n$, 4th column) to a low-pass activity profile rather than a peaked one. We used $\alpha = n/2$, to ensure that the peak scale matched the edge blur. The filtering scheme proposed by Field (1987, Field & Brady, 1997)—in which all channels have the same peak sensitivity to their preferred spatial frequency—is represented here by $\alpha = n$. It exhibits contrast constancy, but does not allow blur coding by peak-finding.

Supplementary methods, with illustrations

Blur mixture experiment

The test edge $I_{\text{mix}}$ consisted of two Gaussian edges superimposed at the same location, with the same polarity, but different blurs ($b_1$, $b_2$):

$$I_{\text{mix}}(x) = l_0\left(1 + c_1\left[2\Phi(x - x_0, b_1) - 1\right] + c_2\left[2\Phi(x - x_0, b_2) - 1\right]\right),$$

where $l_0$ is the fixed mean luminance, $x_0$ is the position of the edge, $\Phi(x, b)$ is the integral of the unit-area Gaussian $G(x, b)$ with space constant (blur) $b$; $c_1$, $c_2$ are the contrasts of the two component
edges. Both edges were at the centre of the screen \((x_0 = 0)\), and overall contrast \(c_1 + c_2\) was constant (0.3).

![Illustration 1](image)

Illustration 1.

The ratio of contrasts of the two component edges \(r = c_1/c_2\) was 0.1, 1, 3, 10, 30 or 100 (shown above, left to right) and the component blurs \((b_1, b_2)\) were (15, 5) or (30, 10) arcmin. Test intervals were 300 msec, separated by a grey (mean luminance) interval of 300 msec. The comparison image in all experiments was a Gaussian edge of contrast \(c = 0.3\), whose blur \(b_0\) was varied by the staircase routine:

\[
I_{\text{gauss}}(x) = I_0 \left(1 + c \cdot [2\Phi(x, b_0) - 1]\right).
\]

![Mixture of 2 Gaussian edges](image)

Illustration 2.

The test luminance waveform is illustrated above for 3 values of the contrast ratio.
Sharpened edge experiment

The test edges were modified ('sharpened') versions of a Gaussian edge whose original blur was \( b = 10, 20 \) or \( 30 \) arcmin. The local contrast function \( C(x) = (I(x) - I_0)/I_0 \) of the original Gaussian edge was defined by \( C(x) = [2\Phi (x, b) - 1] \). The waveform \( C(x) \) (range \(-1\) to \(+1\)) was passed through a Naka-Rushton nonlinear transformation \( C(s + |C|) + \) and after appropriate scaling of amplitude the luminance profile of the modified edge was

\[
I_{\text{sharp}}(x; s) = I_0 \left( 1 + \frac{cC(x)}{s + \text{abs}(C(x))}, (s + \max(C(x))) \right).
\]

Illustration 3.

Values of \( s \) were \([0.1, 0.3, 1, 3, 10, 100, 1000]\). The first 6 are illustrated above, left to right, and three of the luminance waveforms are shown below. The maximum gradient increased as \( s \) decreased, but Michelson contrast \((c = 0.3)\) was constant.
Illustration 4.

**Sine-wave edge experiment**

Test waveforms were a half-period edge, single-period edge, and a sine-wave grating filling the 5 deg display aperture – see Figure 4 in the paper. Six spatial frequencies were tested, ranging from 0.354 to 2.0 c/deg, in half-octave steps (with corresponding half-periods from 84.8 to 15 arcmin). Michelson contrast was 0.32.

**Gaussian-derivative experiment**

Luminance waveforms of the test images were defined as odd-order (−1, 1, 3, 5) Gaussian derivative profiles in the x-direction (shown below) with a flat profile in the y-direction (inset to Figure 4D) at 3 scales (5.7, 11.3, 22.6 arcmin). Order −1 represents the Gaussian integral. Michelson contrast was 0.32. Data are the geometric means of 3 subjects (M.A.G., T.C.A.F., T.S.M.) ± 1 s.e..

[Image of Gaussian derivative profiles]

Illustration 5.

**Length experiment**

Test edge profiles were Gaussian integrals and 4 test blurs were used (2.8, 5.7, 11.3, 22.6 arcmin). Length of the test edge was truncated sharply and symmetrically about the centre of the circular 5 deg test window, as shown below. Comparison edge was 5 deg long, filling the display window as usual.
Illustration 6.

**Edge location experiment (Hesse & Georgeson, 2005)**

Subjects used a short, thin, red cursor line to mark the perceived location of edges across a family of vertical 1-D images differing in phase and blur. The starting point for these images was the $1/f$ amplitude spectrum of a square-wave grating, where $f$ represents the odd-numbered harmonics. New images were created by setting the phase of all the sine-wave Fourier components to the same value $\Phi$. As $\Phi$ increases, the waveform (inset, Figure 4F) changes from bar-like ($\Phi = 0$) to edge-like ($\Phi = 90^\circ$). At $\Phi = 45^\circ$ or $135^\circ$ the waveform is a mixture of the bar and edge profiles. Five levels of blur were created by multiplying the Fourier amplitudes by a Gaussian function, $\exp(-f^2/(\sigma_b^2))$ for $\sigma$, where increasing $\sigma_b$ represents sharper images. The values of $\sigma_b$ (1, 2, 4, 8, 16 c/deg) corresponded to spatial Gaussian blurs of 9.6, 4.8, 2.4, 1.2 and 0.6 arcmin respectively. Data (Hesse & Georgeson 2005) re-plotted in Figure 4F for phases $\Phi = 0$ to 1350 were averaged across 6 observers, 2 r.m.s contrast levels (12%, 24%) and across pairs of stimuli that differed only in contrast polarity (ie. $\Phi$ and $\Phi + 180^\circ$).

Supplementary References
