Predictive Mathematical Models for the Spread and Treatment of Hyperoxia-induced Photoreceptor Degeneration in Retinitis Pigmentosa

IOVS — Supplementary Material

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Figure S1: Simulation results showing exemplar initial oxygen and photoreceptor profiles (see Eq. (7)). The initial oxygen profile is the steady-state distribution corresponding to the initial photoreceptor profile. Panel (a) shows healthy oxygen and photoreceptor profiles without photoreceptor degeneration ($F(\theta, \phi) = 1$). Panel (b) shows oxygen and photoreceptor profiles where photoreceptors have been removed from an annular region with inner and outer boundaries $(\theta_1, \theta_2) = (0.4, 0.6) \times \Theta$ (rad) ($F(\theta, \phi) = \frac{1}{2} (\tanh(S(\theta_1 - \theta)) + \tanh(S(\theta - \theta_2)) + 2)$). Panel (c) shows oxygen and photoreceptor profiles where photoreceptors have been removed from a disc-shaped region with centre eccentricity $\theta_c = 0.3 \times \Theta$ (rad) and radius parameter $\psi = 0.05 \times \Theta$ (rad) ($F(\theta, \phi) = \frac{1}{2} (\tanh(S((\theta - \theta_c)^2 + (\phi - \phi_c)^2 \sin^2(\theta - \psi^2)) + 1))$. See Tables 2 and 3 for the remaining parameter values.
Figure S2: Simulation results showing how retinal degeneration progresses following the loss of an annulus of photoreceptors. Heat maps show the photoreceptor distribution in the initial- (left) and steady-states (right), while $\theta_1$ (rad) and $\theta_2$ (rad) are the initial eccentricities of the inner and outer boundaries of the degenerate annulus respectively. (a) $(\theta_1, \theta_2) = (0.4, 0.6) \times \Theta$ (rad): the degenerate annulus does not expand, (b) $(\theta_1, \theta_2) = (0.08, 0.4) \times \Theta$ (rad): the degenerate annulus expands centrally only, (c) $(\theta_1, \theta_2) = (0.6, 0.9) \times \Theta$ (rad): the degenerate annulus expands peripherally only and (d) $(\theta_1, \theta_2) = (0.08, 0.9) \times \Theta$ (rad): the degenerate annulus expands both centrally and peripherally. See Tables 2 and 3 for the remaining parameter values.