Supplementary material for:

ESTIMATING THE CRITICAL DURATION FOR TEMPORAL
SUMMATION OF STANDARD ACHROMATIC PERIMETRIC STIMULI

Pádraig J. Mulholland¹², Tony Redmond³, David F. Garway-Heath¹, Margarita B. Zlakova², Roger S. Anderson¹²

¹National Institute for Health Research (NIHR) Biomedical Research Centre, Moorfields Eye Hospital NHS Foundation Trust and UCL Institute of Ophthalmology, London, United Kingdom.
²Vision Science Research Group, School of Biomedical Sciences, University of Ulster, Coleraine, Northern Ireland, United Kingdom.
³School of Optometry and Vision Sciences, Cardiff University, Cardiff, United Kingdom.
Methods of analysis for estimating the critical duration

Within the temporal summation literature, a number of different methods of analysis have been used to estimate the critical duration. The methods previously used include (1) manual estimation, (2) constrained least-squares regression, (3) extrapolated intersection analysis and (4) iterative two-phase regression analysis. These methods each assume that a region of complete summation is found for stimuli of short duration. They do, however, differ in that constrained least squares and extrapolated intersection analysis assume no summation to be exhibited for long duration stimuli, whilst the others do not.

The appropriateness of different methods of fitting temporal summation data has received little attention in published literature, with only one previous study suggesting that the choice of analysis may influence the estimate of the critical duration. In this study we estimated the critical duration using each of the four methods previously described in the literature, to determine if (a) the choice of analysis method has a significant influence on critical duration estimates and (b) if, when estimated using the same analysis technique as those used in previous studies, similar values for the critical duration are found. Previously used techniques are described below.

1. Manual estimation

In many early studies, the critical duration was estimated manually from a plot of log contrast threshold against log stimulus duration (e.g. Barlow, Wilson). As part of this analysis, the point at which the summation function deviates from a slope of -1 was defined as the critical duration. Critical duration values were estimated manually from our temporal summation functions by one of the authors (PJM) and a naïve, untrained analyst. To exclude researcher bias, summation functions were firstly pseudo-
anonymised and randomized. This method is depicted schematically in figure 1 in the main manuscript.

2. Constrained least squares regression analysis

This method was described by Krauskopf and Mollon\textsuperscript{4} and Mitsuboshi et al.\textsuperscript{5} for the analysis of temporal summation in the colour-opponent pathways. It has subsequently been used in a number of other studies, including those investigating temporal summation in ocular disease.\textsuperscript{6, 7} The method fits two lines of constrained slope to selected data points in a given data set using a least squares method. Initially contrast thresholds (log $\Delta I/I$) were plotted as a function of stimulus duration (log(t)). The first line ($L_1$) in the analysis, with a slope of -1, was then fitted to the three shortest stimulus durations (<30 msec) where it is assumed complete summation occurs (Figure 1). The second line ($L_2$) was constrained to have a slope of zero and was applied to stimuli of duration greater than 100 msec (in this study two data points) where summation is assumed to be incomplete. The point at which the two lines met was taken to represent the critical duration.

3. Extrapolated intersection analysis

This method, proposed by Funkhouser and Fankhauser,\textsuperscript{8} introduces a slight amendment to the constrained regression analysis previously described. Like the least-squares analysis, it is assumed complete summation occurs for a range of short duration stimuli (the authors propose below 17.8 msec). A line with a slope of -1 was used to describe the trend of the data points in this region. It is also suggested that the slope ($m$) of the first
line may be empirically determined by local linear regression analysis of the data points that lie along this line, to improve accuracy. As only two stimuli with durations shorter than 17.8 msec were used in the current study, regression analysis was not performed and a slope value of -1 was assumed for the first line. Once the slope values were determined for both lines, the critical duration was estimated by calculating the extrapolated intersections for four combinations of the shortest (8.3, 16.5) and longest (124, 198.3 msec) stimulus durations. This was performed using equation 1 where \( t_s \) is one value (duration in msec) from the group of short stimulus durations, \( \Delta I_l \) a contrast threshold (in log units) for a given long duration stimulus and \( \Delta I_s \) a contrast threshold (in log units) for the selected short duration stimulus \( t_s \).

\[
t_c = t_s \times \exp[\log(10) \times (\Delta I_s - \Delta I_l)]
\]  

(1)

A total of four critical duration estimates were produced using this method. The final value for the critical duration was calculated as the mean of all individual critical duration estimates with an associated standard deviation (SD) value. This method is described schematically in figure 1.

4. Iterative two-phase regression analysis

Two-phase regression analysis\(^9\) (Levenberg-Marquardt estimation) was performed for all test locations using MATLAB and the Statistics toolbox. In this analysis the slope of the first line was constrained to -1, in line with complete temporal summation. The slope and intercept of the second line was, however, free to vary. Prior to performing this analysis,
an estimate of the breakpoint \( (b_p) \), second-line slope \( (m_p) \) and second-line intercept \( (i_p) \) were input to the following function:

\[
[i, m, t_c] = i_p - (1 \times t) + m_p \times (t - b_p) \times (t > b_p)
\]  

The final position of both lines in the analysis, along with the slope \( (m) \), intercept \( (i) \) of the second line and critical duration \( (t_c) \), was finally determined by least-squares analysis. The maximum number of iterations was set at 1000 for all analysis. The point at which the two lines in the model meet (breakpoint) was taken to represent the critical duration estimate (Figure 1).

**References**