Supplementary Material

Effect of pressurization on the flat punch indentation measurement

The effect of pressurization on the stiffness of an ellipsoid with point-load indentation was studied by Lazarus et al. The stiffness from point-load indentation is shown to increase with pressure. In our study, the indentation is not point indentation, but flat punch indentation. Eq.(1) in our analysis presumed that the effect of pressurization of the ellipsoidal shell on the tangent modulus measurement is negligible. The negligible contribution of pressure to the tangent modulus is shown in this supplement using silicone shell models.

The details of the fabrication of the silicone shell models was described by Ko et al. Three silicone rubber spherical shell models, with different radii of curvature $R$ and thicknesses $t$ ($R=10.60\text{mm}, t=1.45\text{mm}$; $R=10.37\text{mm}, t=0.98\text{mm}$; and $R=9.69\text{mm}, t=0.52\text{mm}$), were fabricated for the indentation tests. The silicone models were cannulated by a hypodermic needle and the internal pressure (varied between 7.9 mmHg and 39.3 mmHg) of the models was controlled using a manometer. The tangent modulus of the silicone models was determined using Eq.(1) from indentation data obtained with flat punch indenters tip diameters 1.7 mm, 2.0 mm, 3.0 mm, 4.3 mm and 5.1 mm. The in-plane stress $\sigma$ was calculated using Eq.(5).

For point indentation, Lazarus et al found that the stiffness of an ellipsoidal shell
varies according to \[ \frac{\pi/2(\tau^2 - 1)^{1/2}}{\arctanh\left(1 - \tau^{-2}\right)^{1/2}}, \]
where \( \tau = pR^2/(4EDt)^{1/2} \),

\[ D = Et^3/12(1 - v^2) \]
and \( p \) is the internal pressure. To study if the flat punch indentation in this paper would have similar dependence on the measurement, Eq.(1) was rearranged and multiplied by the same factor,

\[ S = \frac{Et^2}{a(R - t/2)\sqrt{1 - v^2}} \frac{\pi/2(\tau^2 - 1)^{1/2}}{\arctanh\left(1 - \tau^{-2}\right)^{1/2}}, \tag{S1} \]

where the tangent modulus \( E \) can be determined numerically.

For comparison, the tangent modulus of the silicone rubber material was also determined using standard three-point bending test (MTS 642.001) with bars (32.0 mm x 18.6 mm x 9.9 mm) fabricated from the same material. The stress \( \sigma \) and tangent modulus \( E \) were determined from the test data using the beam bending equations,

\[ \sigma = \frac{3FL}{2bh^2} \tag{S2} \]
and

\[ E = \frac{L^3}{4bd^3} \frac{F}{\delta} \tag{S3} \]

where \( L \) is the support span, \( b \) is the width and \( d \) is the thickness of the bar.

The tangent moduli calculated using Eqs.(1), (S1) and (S3) were compared. The results are shown in Figure S1. The results from Eq.(S1) is shown to deviate from the 3-point bending test results, and that the pressure dependence of the flat punch
indentation cannot be described by the point-load indentation equation. Figure S1 showed that the tangent modulus calculated using Eq.(1) is in good agreement with the results from the standard 3-point bend test. This means that the flat punch indentation in this study is independent of the internal pressure, pressure effect can be ignored in Eq.(1).

References